



WROCLAW UNIVERSITY OF ENVIRONMENTAL AND LIFE SCIENCES

GNSS-based estimation of slant total delay towards satellite

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Schedule

- 1. Very short introduction to GNSS phase observables and STD**
- 2. Overview of processing methods used for STD estimation**
- 3. Examples of slant total delay estimation (literature review),**
- 4. Example from PPP procedure,**
- 5. Conclusion**

GNSS phase observable and STD

Troposphere is non-dispersive for electromagnetic waves up to 15GHz. The GNSS signals (1,176 – 1,602 GHz) are refracted (delayed) in the same way. Any phase observable Φ_r^s from satellite s to receiver r (so-called „zero-differenced” observable) may be expressed as:

$$\Phi_r^s = \rho + c(t_r - t^s) + STD + ION + MP + APCd + v$$

Where:

ρ	is geometric distance from satellite to receiver,
$c(t_r - t^s)$	is linear value of satellite and receiver clock errors,
STD	is slant troposphere delay of GNSS signal,
ION	is impact of ionosphere on GNSS signal frequency,
MP	is phase multipath effect,
$APCd$	is antenna phase center residual delay,
v	is unmodelled residual noise.

The STD is then:

$$STD + v = \Phi_r^s - (\rho + c * (t_r - t^s) + ION + MP + APCd)$$

Removing the non-troposphere impact on signal

$$STD + v = \Phi_r^s - (\rho + c * (t_r - t^s) + ION + MP + APCd)$$

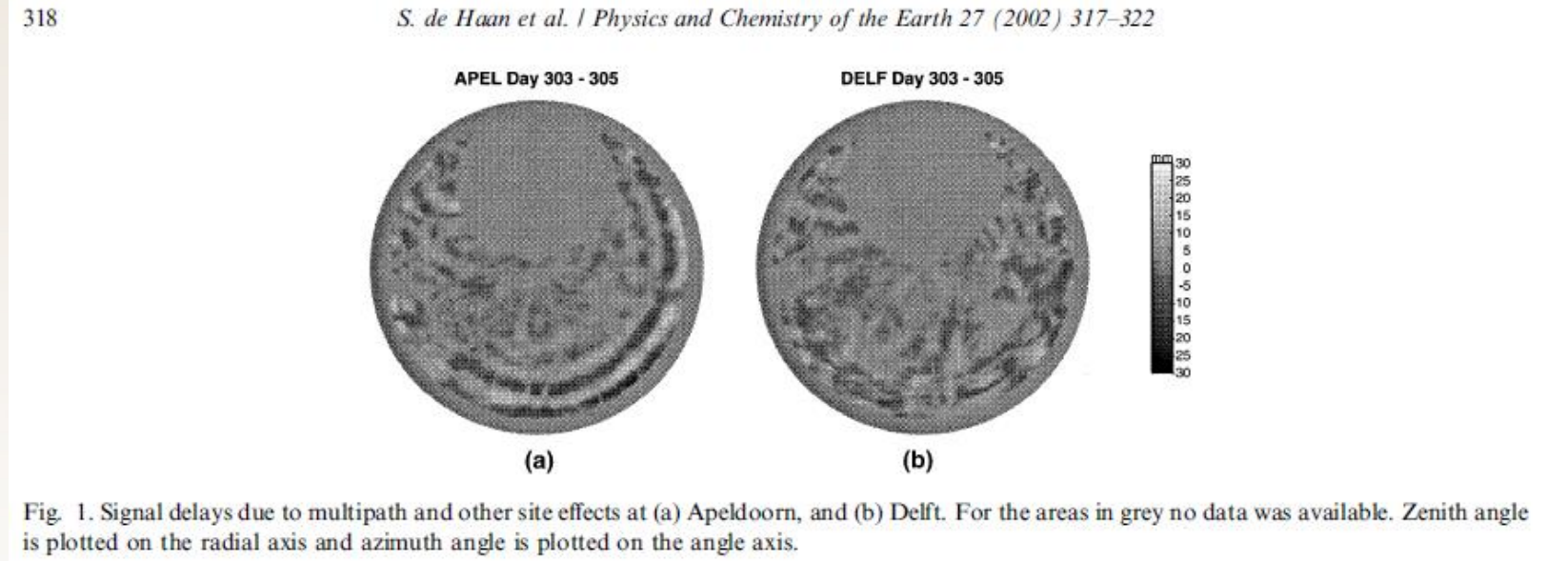
- ρ geometric distance contains the coordinates of satellite and receiver. Satellite coordinates error is reduced by introducing precise orbits or cancelled during the double differencing of phase observables, receiver error is estimated in zero-differenced processing or cancelled in double-differenced processing.
- $c(t_r - t^s)$ Satellite clock error is reduced by introducing precise highrate clocks or cancelled during the double differencing of phase observables, receiver clock error is estimated in zero-differenced processing or cancelled in double-differenced processing,
- ION impact of ionosphere on GNSS signal is cancelled by combining L_1 and L_2 frequencies in ionosphere-free linear combination (L_3). Higher order ionosphere correction may be calculated from model or estimated,

Removing the non-troposphere impact on signal

$$STD + v = \Phi_r^s - (\rho + c * (t_r - t^s) + \cancel{ION} + \cancel{MP} + \cancel{APCd})$$

MP

phase multipath effect, may be reduced by mapping the effect at each processed station,



S. de Haan, H. van der Marel, S. Barlag (2002). Comparison of GPS slant delay measurements to a numerical model: case study of a cold front passage. Physics and Chemistry of the Earth 27 (2002) 317–322

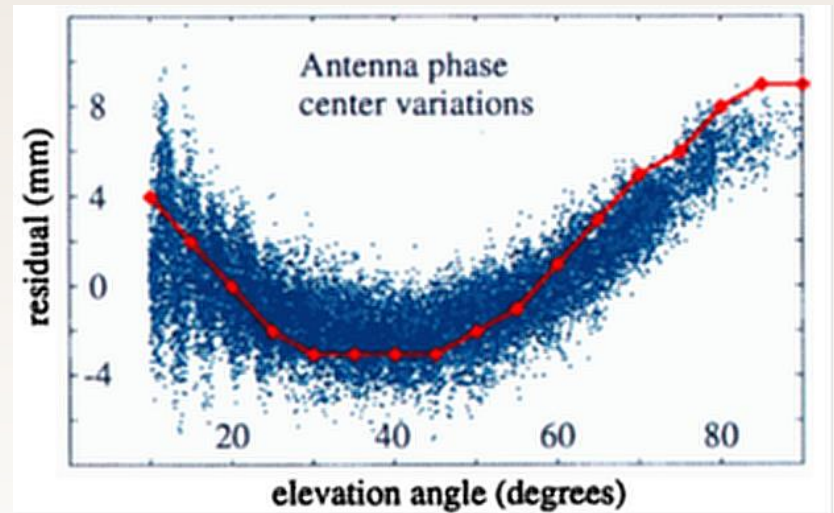
Removing the non-troposphere impact on signal

$$STD + v = \Phi_r^s - (\rho + c * (t_r - t^s) + \cancel{ION} + \cancel{MP} + \cancel{APCd})$$

APCd

Satellite and receiver antenna phase center model is usually assumed to be known and eliminated using the antenna phase center absolute model. If individual calibration model for each antenna is not provided, the residual delay after removing all other effects can be estimated.

**Example of residual antenna delay
estimated w.r.t. radiometer STD**



Chris Alber, Randolph Ware, Christian Rocken, John Braun (2000). Obtaining single path phase delays from GPS double differences. *Geophysical Research Letters*, vol. 27, no. 17, pages 2661-2664, September 1, 2000

STD estimation model

The Slant Total Delay (STD) caused by refraction in neutral atmosphere may be divided to parts: hydrostatic (dry) and non-hydrostatic (wet). As an effect we obtain Hydrostatic Delay (HD) and Wet Delay (WD):

$$STD = \int (n - 1) ds = 10^{-6} \int N_{dry} ds + 10^{-6} \int N_{wet} ds = SHD + SWD$$

where n is a refractivity index and N is refractivity (eg. Essen and Froome 1951)

$$STD(t, a, z) = ZTD_{apr}(t) * mf(z) + dZTD(t) * mf(z) + G_N(t) * \frac{\partial mf}{\partial z} \cos(a) + G_E(t) * \frac{\partial mf}{\partial z} \sin(a)$$

A priori model

*Estimated
ZTD correction*

*Estimated
Horizontal ZTD gradients*

$$STD(t, a, z) = ZHD_{apr}(t) * mf_{Dry}(z) + ZWD_{est}(t) * mf_{Wet}(z) + G_N(t) * \frac{\partial mf}{\partial z} \cos(a) + G_E(t) * \frac{\partial mf}{\partial z} \sin(a)$$

STD estimation: zero-difference (Bender et al., 2011)

STD estimation implemented in EPOS software (developed at GFZ). The PPP method is used to estimate the coordinates, troposphere parameters and epoch-wise estimation of satellite and clock biases. The a priori ZTD model is Saastamoinen (1972) with GMF mapping functions (Boehm et al., 2006).

$$STD = mf_{DryGMF} * ZHD + mf_{WetGMF} * [ZWD + \cot(z) * (G_N \cos(\phi) + G_E \sin(\phi))] + \delta$$

Where t is time epoch, a is azimuth, z is the zenith angle, G_N , G_E are the horizontal gradients, ϕ is the geographic latitude, δ is the post-fit phase residual from PPP method

and

$$\frac{\partial mf_{WetGMF}}{\partial z} \cos(a) = \cot(z) \cos(\phi), \quad \frac{\partial mf_{WetGMF}}{\partial z} \sin(a) = \cot(z) \sin(\phi).$$

*Michael Bender, Galina Dick, Maorong Ge, Zhiguo Deng, Jens Wickert, Hans-Gert Kahle, Armin Raabe, Gerd Tetzlaff, (2011). **Development of a GNSS water vapour tomography system using algebraic reconstruction techniques.** Advances in Space Research 47 (2011) 1704–1720;*

STD estimation: zero-difference (Bender et al., 2011)

Requirements of the method:

- Precise satellite clock and orbit is essential in this method,
- Ambiguity resolution may increase the accuracy,
- Maps of multipath effect and antenna phase delay are required (but not mentioned in the paper).

Advantages:

- Zero-differenced processing is faster than double-differenced,
- Easy applicable to the software working in zero-differenced mode,
- Can work in near real-time.

Important remark by:

Lei YANG, Chris HILL and Terry MOORE (2013). Numerical weather modeling-based slant tropospheric delay estimation and its enhancement by GNSS data. Geo-spatial Information Science, Vol. 16, No. 3, 186–200, <http://dx.doi.org/10.1080/10095020.2013.817107>

„The gradient terms are solved as extra unknowns in the PPP solution. Although they can absorb the troposphere profile asymmetry to a certain extent, this absorption is limited by its linear-plan modelling, and cannot fully describe the complicated azimuth dependent STD variation. As these two gradient terms are solved together with coordinates, they will also absorb some other non-tropospheric variations.”

STD estimation: zero-difference (patent)

Xiaoming Chen (Trimble Navigation Limited) was granted the patent for GNSS atmospheric estimation with federated ionospheric filter. International Patent WO 2010/021656 A2 dated 25 February 2010 (TNL A-2526PCT);

The ionosphere-free carrier phase observation is written as:

$$L_c = \rho + \mathbf{c} * (\mathbf{t}_r - \mathbf{t}^s) + ZTD * mf(z) + G_N(t) * \frac{\partial mf}{\partial z} \cos(a) + G_E(t) * \frac{\partial mf}{\partial z} \sin(a) + N_c + v$$
$$\hat{N}_c = N_c + \Delta_c^r - \Delta_c^s$$

With the network-fixed ambiguities \hat{N}_c , where Δ_c^r and Δ_c^s are respectively the receiver and satellite dependent biases in the ionosphere-free undifferenced ambiguities, the ambiguity-reduced ionosphere-free carrier phase observation becomes:

$$\check{L}_c = L_c - \hat{N}_c$$
$$\check{L}_c = \rho + \mathbf{c} * (\mathbf{t}_r - \mathbf{t}^s) + ZTD * mf(z) + \Delta_c^r - \Delta_c^s + G_N(t) * \frac{\partial mf}{\partial z} \cos(a) + G_E(t) * \frac{\partial mf}{\partial z} \sin(a) + v$$
$$\check{L}_c = \rho + \mathbf{c} * ((\mathbf{t}_r - \Delta_c^r) - (\mathbf{t}^s - \Delta_c^s)) + ZTD * mf(z) + G_N(t) * \frac{\partial mf}{\partial z} \cos(a) + G_E(t) * \frac{\partial mf}{\partial z} \sin(a) + v$$

The terms Δ_c^r and Δ_c^s are absorbed by the new satellite and receiver clock error terms $\check{\mathbf{t}}_r$ and $\check{\mathbf{t}}^s$:

$$\check{L}_c = \rho + \mathbf{c} * ((\check{\mathbf{t}}_r - \check{\mathbf{t}}^s)) + ZTD * mf(z) + G_N(t) * \frac{\partial mf}{\partial z} \cos(a) + G_E(t) * \frac{\partial mf}{\partial z} \sin(a) + v$$
$$STD = \check{L}_c - \rho + \mathbf{c} * ((\hat{\mathbf{t}}_r - \hat{\mathbf{t}}^s))$$


Where $\hat{\mathbf{t}}_r$ and $\hat{\mathbf{t}}^s$ are the estimates of $\check{\mathbf{t}}_r$ and $\check{\mathbf{t}}^s$.

STD estimation: zero-difference (patent)

Xiaoming Chen (2010). **GNSS atmospheric estimation with federated ionospheric filter.** Trimble Navigation Limited. International Patent WO 2010/021656 A2 dated 25 February 2010 (TNL A-2526PCT);

Presented method is implemented to the Trimble Pivot® software

TRIMBLE ATMOSPHERE APP TECHNICAL NOTES

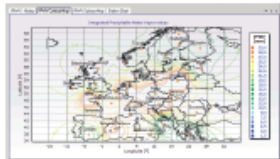
 The Trimble® Atmosphere App is a unique scientific application designed to support atmospheric estimation based on GNSS and weather station data. The Trimble Atmosphere App operates on the Trimble Pivot™ platform, the foundation for multiple infrastructure apps that serve various markets and their specific needs.

TRIMBLE ATMOSPHERE APP

Designed to meet the specific needs of Earth Systems applications, the Trimble Atmosphere App enables users to evaluate ionosphere and troposphere conditions. Atmosphere Watch and Weather Condition are the modules within the application that allow users to calculate a variety of different atmospheric parameters:

- Integrated Precipitable Water Vapor (IPWV)
- Total Electron Content (TEC)
- Tropospheric Slant Delay and Wet Slant Delay

Advanced modern algorithms are used to derive atmospheric state parameters from GNSS observables and meteorological sensor data. Both postprocessing and real-time approaches are supported. The Trimble Atmosphere App allows you to present results in ASCII or binary (BINARY) format or in graphical form. In addition to station or condition charts, results can be visualized as a contour or a surface map.



Trimble Atmosphere App Contour Map

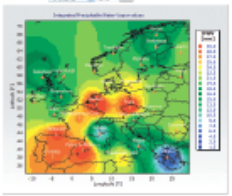
TRIMBLE ATMOSPHERE APP PAGES IN PIVOT WEB APPLICATION

The Trimble Pivot platform includes a basic web installation. The Trimble Atmosphere App adds the following web pages to support and display the atmospheric conditions to a wide user range.

- IPWV Map
- Station Chart
- Condition Chart
- IPWV Contour Map, Surface Map and Surface Map Animation
- TEC Contour Map, Surface Map and Surface Map Animation
- TEC Surface Map, Surface Map and Surface Map Animation
- TEC Surface Map Animation

The content of these pages are defined through a management page. In addition, all visualizations within the Web Application are interactive, to enable the user to zoom in on a detail or select a specific time from which the user would like to receive the information.

The Sensor Map provides an overview of the network layout with health and status information, while the IPWV map shows current water vapor information per station.



Trimble Atmosphere App IPWV Surface Map

TRIMBLE ATMOSPHERE APP STATUS VIEW


The Trimble Atmosphere App Status View provides an overview of the atmosphere watch processors in the network as well as available weather condition modules. The details pane provides the settings on the different modules as well as the TEC and IPWV results. The App Status View assists the operator in analyzing critical and non-critical situations in real-time to better assist field end users. This smart and interactive feature will point you to the exact module and functionality you want to analyze. The traffic light indicator provides a high level application status summary.

ORDER INFORMATION

Default Configuration	Part Number
Trimble Atmosphere App	06805-00

Optional Functions*	Part Number
Water Vapor (IPWV) calculation	06805-01

*May be included for an additional charge



STD estimation example: zero-difference

STDs were calculated here using the Bender et al. (2011) method:

$$STD = m f_{DryGMF} * ZHD + m f_{WetGMF} * [ZWD + \cot(z) * (G_N \cos(\phi) + G_E \sin(\phi))] + \delta$$

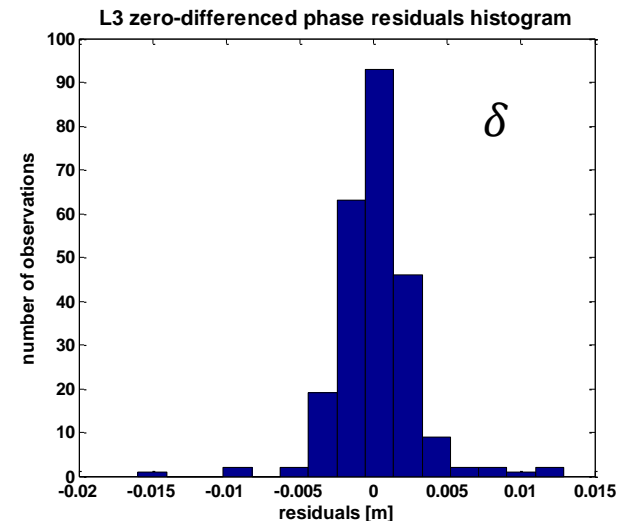
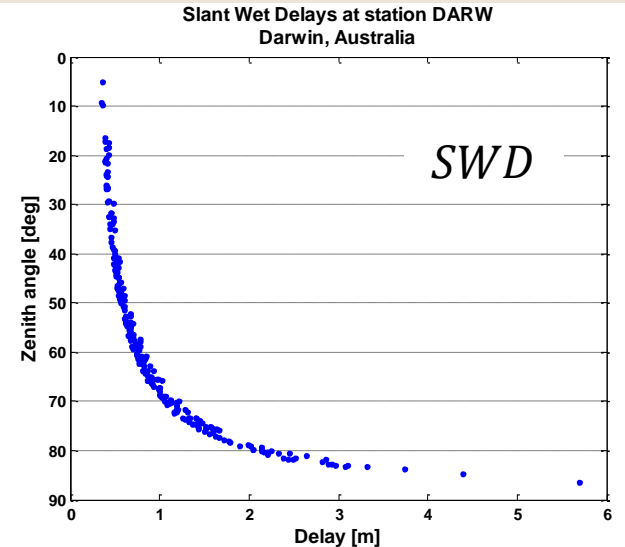
and Bernese GNSS Software 5.2 troposphere estimates (TRP)

Y	M	D	H	MM	S	Model	Corr	mZTD	ZTD	GE	mGE	GN	mGN
2011	04	06	00	00	00	2.2748	0.38893	0.00076	2.66373	0.00029	0.00005	0.00025	0.00007

and phase zero-differenced residuals (RES -> FRS)

Num	Epoch	Frq	Sat.	Phase residual Value	Elev	Azi
1	1	3	1	0.202153357997348487D-02	15.75	213.46

$$SWD = m f_{WetGMF} * [ZWD + \cot(z) * (G_N \cos(\phi) + G_E \sin(\phi))] + \delta - m f_{DryGMF} * ZHD$$



STD estimation: double-difference (Alber et al., 2000)

Double-differenced processing have great advantages over the zero-difference processing. These are: cancelling the satellite orbit, satellite and receiver clock errors, easy ambiguity resolution. The idea of Alber et al., (2000) was to calculate back the error free zero-differenced phase observables from ambiguity free double-differences and compute the residual phase observation which reflect the troposphere anisotropy.

$$\begin{aligned} s_{AB}^1 &= \Phi_A^1 - \Phi_B^1, & \text{single-difference (receivers A B, satellite 1),} \\ s_{AB}^2 &= \Phi_A^2 - \Phi_B^2, & \text{single-difference (receivers A B, satellite 2),} \\ dd_{AB}^{12} &= s_{AB}^1 - s_{AB}^2, & \text{double-difference (receivers A B, satellites 1 2)} \end{aligned}$$

To convert double differences to single differences, the double differences **dd** are written as the product of a matrix **D** and a vector of single differences **s**,

$$Ds = dd$$

The matrix **D** cannot be inverted, because for **n** single differences we have **n – 1** independent double-differences. We must then introduce the additional constraint for at least one of the single differences, then the matrix **D** is easily invertible. If the final post-fit double-differences are used, the assumption that $\sum w_i s_{AB}^1 = 0$ may be taken, and single differences **s** may be estimated.

$$\begin{bmatrix} w_1 & w_2 & w_3 & \cdots & w_n \\ 1 & -1 & 0 & \cdots & 0 \\ 1 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & -1 \end{bmatrix} \begin{bmatrix} s_{AB}^1 \\ s_{AB}^2 \\ s_{AB}^3 \\ \vdots \\ s_{AB}^n \end{bmatrix} = \begin{bmatrix} \sum w_i s_{AB}^1 \\ dd_{AB}^{12} \\ dd_{AB}^{13} \\ \vdots \\ dd_{AB}^{1n} \end{bmatrix}$$

Chris Alber, Randolph Ware, Christian Rocken, John Braun (2000). **Obtaining single path phase delays from GPS double differences.** *Geophysical Research Letters*, vol. 27, no. 17, pages 2661-2664, September 1, 2000

Braun, J., Rocken, C., Ware, R. (2001). **Validation of line-of-sight water vapor measurements with GPS.** *Radio Sci.* 36 (3), 459–472, 2001.

STD estimation: double-difference (Alber et al., 2000)

Then the same procedure may be applied to obtain the zero-differences \mathbf{z} to a given satellite with the assumption that $\sum \mathbf{W}_i \mathbf{z}_A^i = \mathbf{0}$ and the weights \mathbf{W} are elevation dependent:

$$D_1 \mathbf{z} = \mathbf{s}_1$$
$$\begin{bmatrix} W_A & W_B & W_C & \cdots & W_n \\ 1 & -1 & 0 & \cdots & 0 \\ 1 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & -1 \end{bmatrix} \begin{bmatrix} z_A^i \\ z_B^i \\ z_C^i \\ \vdots \\ z_Z^i \end{bmatrix} = \begin{bmatrix} \sum W_i z_A^1 \\ s_{AB}^i \\ s_{BC}^i \\ \vdots \\ s_{AZ}^i \end{bmatrix}$$

The \mathbf{z} values represent the slant delay fluctuations about the model used to compute the \mathbf{s} and \mathbf{dd} values. The slant total delay **STD** is then equal to:

$$STD_A^1 = m f_{Dry} * ZHD_A^1 + m f_{Wet} * ZWD_A^1 + z_A^1$$

Requirements of the method:

- Large network will produce better results, because of the values of \mathbf{z} are relative to the ensemble mean of the network, This implies the need of careful network processing to minimize biases or introduction of absolute **STD** to lever the solution,
- Multipath error map should be calculated for each processed station

STD estimation: double-difference (Alber et al., 2000)

Advantages:

- Method is based on data almost free from satellite, orbit, satellite/receiver clocks, and ionosphere effects on STD estimation,
- Easy applicable to the Bernese GNSS Software,
- Can work in near real-time
- The accuracy estimated during the tests over 3-day period is ~ 2 mm in terms of agreement between GNSS derived SWD and WV Radiometer data. Braun et al. (2001) obtained using this method the elevation dependent accuracies of SWD from 1.4 for zenith to 9.1 mm at low elevations.

Disadvantages:

- The zero-mean assumptions $\sum w_i s_{AB}^1 = \mathbf{0}$ and $\sum W_i z_A^i = \mathbf{0}$ may not be true and will lead then to biased results,
- The constraints applied to the solution must come from independent sources and have good quality (eg. Water Vapor Radiometers).

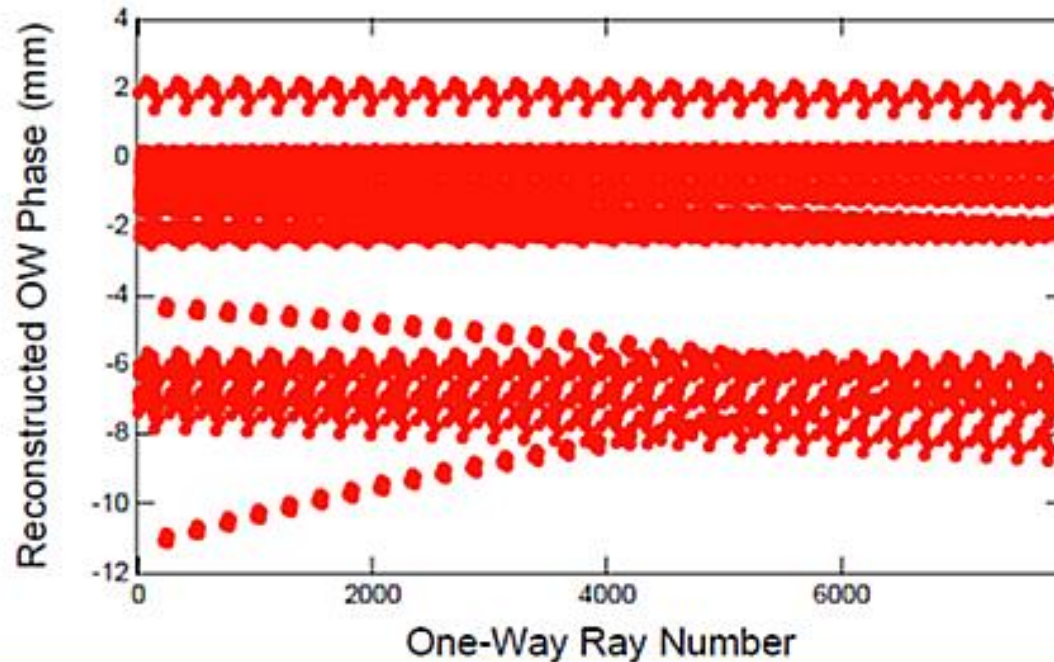
Pedro Elosegui and James I. Davis (2003). Feasibility of directly measuring single line-of-sight GPS signal delays. Smithsonian Astrophysical Observatory.

Taking into account the disadvantages mentioned above, Pedro Elosegui and James Davis (2003) on the basis of the simulated data revealed that using the Alber et al. (2000) method the anisotropies in the atmosphere will cause wrong reconstructed zero-differences and the expected improvement in STD estimation will be lost within the magnitude of the error of reconstruction.

STD estimation: double-difference (Alber et al., 2000)

Pedro Elosegui and James I. Davis (2003). *Feasibility of directly measuring single line-of-sight GPS signal delays*. Smithsonian Astrophysical Observatory.

Reconstruct Atmosphere



All seven regional sites recover ~9 mm of the 10 mm difference between the anisotropy ray and the rest of rays, but all its reconstructed phases are biased. The reconstructed phases of all other sites are also biased up to the 3 mm level, except for the distant sites, which for the low elevation-angle satellites, show biases at the 4–6 and 7–11 mm level.

Conclusions

1. The zero-differenced STD estimation technique is the most promising, when real-time satellite precise orbits and clocks are available. It is also easy to implement to any GNSS PPP processing software.
2. Double-differenced method with inversion to zero-differenced phase observables needs more real-life tests and improvement in constraining. Can be used where zero-difference solution cannot be done (without precise satellite clocks),
3. The ways to improve the STD estimation are:
 - Development of new mapping functions (e.g. from raytracing), especially for low elevations or selected areas,
 - Increase the number of observations by multi-GNSS processing,
 - Own estimation of clocks and biases,
 - Separation of non-troposphere effects from SWD.

Thank You for attention! jan.kaplon@igig.up.wroc.pl

Bibliography

- Chris Alber, Randolph Ware, Christian Rocken, John Braun (2000). Obtaining single path phase delays from GPS double differences. Geophysical Research Letters, vol. 27, no. 17, pages 2661-2664, September 1, 2000*
- Braun, J., Rocken, C., Ware, R. (2001). Validation of line-of-sight water vapor measurements with GPS. Radio Sci. 36 (3), 459–472, 2001;*
- Michael Bender, Galina Dick a, Maorong Ge, Zhiguo Deng, Jens Wickert, Hans-Gert Kahle , Armin Raabe, Gerd Tetzlaff. (2011). Development of a GNSS water vapour tomography system using algebraic reconstruction techniques. Advances in Space Research 47 (2011) 1704–1720;*
- Braun, J., Rocken, C., Ware, R. (2001). Validation of line-of-sight water vapor measurements with GPS. Radio Sci. 36 (3), 459–472, 2001;*
- Xiaoming Chen (2010). GNSS atmospheric estimation with federated ionospheric filter. Trimble Navigation Limited. International Patent WO 2010/021656 A2 dated 25 February 2010 (TNL A-2526PCT);*
- Pedro Elosegui and James I. Davis (2003). Feasibility of directly measuring single line-of-sight GPS signal delays. Smithsonian Astrophysical Observatory.*
- S. de Haan, H. van der Marel, S. Barlag (2002). Comparison of GPS slant delay measurements to a numerical model: case study of a cold front passage. Physics and Chemistry of the Earth 27 (2002) 317–322;*
- T. Pany (2002). Measuring and modeling the slant wet delay with GPS and the ECMWF NWP model. Physics and Chemistry of the Earth 27 (2002) 347–354;*
- T. Pany, P. Pesec and G. Stangl (2001). Atmospheric GPS Slant Path Delays and Ray Tracing Through Numerical Weather Models, a Comparison. Phys. Chem. Earth (A), Vol. 26, No. 3, pp. 183-188,2001*
- Lei YANG, Chris HILL and Terry MOORE (2013). Numerical weather modeling-based slant tropospheric delay estimation and its enhancement by GNSS data. Geo-spatial Information Science, Vol. 16, No. 3, 186–200, <http://dx.doi.org/10.1080/10095020.2013.817107>*